**Date:** 21 August, 2015

**Experiment No. 1**

**Aim:** To check the consistency of sample mean for a negative exponential and normal distribution.

**Experiment:** **1.** Generate a random sample form negative exponential distribution with parameter λ=5 and check if the sample mean is consistent estimator of population mean or not.

**2.** Generate a random sample form N(3,16) distribution and check if sample mean is a consistent estimator of population mean or not. Also obtain the asymptotic relative efficiency (ARE) of sample median with respect to sample mean and verify the standard result.

**Theory:**

**(Exponential Distribution):** Consider a random variable X ~ exponential (λ). Then pdf is given by

f (x; λ) = ; x>0, λ>0

Its distribution function is given by

F(x) = 1 - ~ U(0,1)

Which gives

X = -λlog(1- u)

We will use this transformation to generate random variables. Also sample mean is given by:

x̅ =

and sample variance is given by:

Variance (x̅) = σ2/n

ARE=

**(Normal Distribution):** Consider a random variable X ~ N(µ, σ2), then its pdf is given by

f (x; µ, σ2) = -∞<x<∞; -∞<µ<∞; σ2 > 0

let u1, u2 Ԑ U(0, 1)

then x1 = and x2 = 2

where x1, x2 ~ N(0,1)

y1 = µ + σx1 and y2 = µ + σx2 ~ N(µ, σ2)

Now a statistics x̅ is a consistent estimator of population mean if

E(X) = µ and var(X) -> 0 as n -> ∞ where µ is the parameter of the distribution.

**Algorithm: (Negative Exponential Distribution)**

1. Open the file “expo1.txt” to read the data and “expoout1.txt” to write the results using pointers.
2. Use randomize function to generate random numbers.
3. Numbers generated will lie between 0 and 1, we will convert them into negative exponential sample using transformation given in theory.
4. Then we will calculate mean and variance of the sample with predefined mean and variance function from “batmean.h” header file.
5. It is expected that as the sample size increases, sample mean should converge to population mean which is given to us.
6. It is also expected that as sample size increases, sample variance should converge to zero. Hence we have obtained consistent estimators.
7. Results are expected in the file “expoout1.txt”.

**(Normal Distribution)**

1. Open the file “norm1.txt” to read the data and “normout1.txt” to write the results using pointers.
2. Use randomize function to generate random numbers.
3. Numbers generated will lie between 0 and 1, we will convert them into nornal sample using transformation given in theory.
4. Then we will calculate mean and variance of the sample with predefined mean and variance function from “batmean.h” header file.
5. It is expected that as the sample size increases, sample mean should converge to population mean which is given to us.
6. It is also expected that as sample size increases, sample variance should converge to zero. Hence we have obtained consistent estimators.
7. Results are expected in the file “normout1.txt”.

**Results:**

1. For negative exponential distribution we see that when the sample size is 5000 we see that sample mean is 4.975409 and sample variance is 0.004930.

Sample Mean Population Mean

Sample Variance 0 0

Hence we conclude that ample mean is a consistent estimator of population mean.

1. For normal distribution we see that when the sample is 2000, the sample mean is 2.077951, sample variance is 0.001106, median is 0.001736 and ARE is 63.694%. This implies that as n approaches ∞ the relative efficiency is 63.694%.

Sample Mean Population Mean

Sample Variance 0 0

**Conclusion:**

In this experiment we have proved that sample mean is a consistent estimator of population mean in the case of negative exponential and normal distribution.